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Introduction

> *Persons attempting to find a motive in this narrative will be prosecuted;*
> *persons attempting to find a moral in it will be banished;*
> *persons attempting to find a plot in it will be shot.*

You will remember that Mark Twain included those words before his novel *Huckleberry Finn.* I suppose something similar could be said about *this* book but substituting the words *rigor*, *completeness* or *pedagogical correctness.*

This book is **not** aspiring to be the universe's authority on introductory physics. It is not even a *self-contained* treatment of the subject. Rather it's a *resource* designed with the hope of making your experience in physics a bit easier.

When a professor writes a textbook, there is a *lot* to be wary of. Typically, the professor is very proud of it. Often they possess some unhealthy expectation that students will parse and ponder each glorious sentence, appreciating and worshiping their wisdom enshrined between two glossy book covers. More than this, once a professor has written a text for a course, some seem to labor with the impression that their text can run their course for them.

But this book only aims to be a *resource* for assisting you in trigonometry-based physics classes. While it's a bit more *focused* and much less *expensive* than our previous text, I'm under no delusion that it is far from perfect. But I *do* believe it's better option than what we were using before.

What Exactly Do You Mean By "Hand-Holding"?

This textbook is *backwards designed* around getting students to be self-sufficient in physics problem-solving strategies. Learning objectives were established *first*, *then* problems sets were developed with those goals in mind. The chapter text, discussion and examples were written last. Physics is a sweeping landscape of topics wherein it is easy to lose direction. If I thought a particular topic did not help students hone their physics problem-solving capability, it was cut from the text. It is hoped that this book will feel more "focused" than off-the-shelf physics books.

At the suggestion of students, this text contains more examples per page than any other physics text of which I am aware. While some texts highlight the importance of the *correct answer*, I emphasize *correctness of process* and *method of approach,* a priority which is reflected strongly between these covers.

As I prepared notes for my PHYS 2010 course the past few years, I stuck them under the working title "Physics By Hand-Holding." It was a joke between students and me, an acknowledgment that we were committing some type of apparent sin making the discipline approachable.

If I buy a textbook, I expect to be *smarter* after reading it. *That* burden is on the *author.* I paid them hundreds of dollars. I did *not* purchase the book to find out how smart the *author* is, or to learn how *difficult* a subject is. A textbook should make me proficient in its subject with the *least hassle possible.* Full stop.

My impatience with stuffy physics textbooks has led to me adopt a comparatively *explicit* teaching style. This is the kind of textbook-writing which some specific *other* authors might term "hand-holding." True, "hand-holding" is sometimes a derogatory expression for teaching in a way which requires little effort from students. But that is not the intent of this book.

The Philosophy of This Text

The primary goal of this book is to build a student's confidence that they (and anyone!) can solve a physics problem. Once that confidence emerges, a student suddenly finds themselves free to appreciate the principles and patterns which motivate a problem-solving technique. In this sense, we *hand-hold.* We walk students through problem-solving strategies step by step, treating simple problems first and then working toward more complex ones.

If the title sounds academically shameless, it is meant to. This particular book for my classes blatantly attempts to teach trigonmetry-based physics more on a *student's* terms. Often as I wrote this text and looked at my photo rosters for my courses, I asked the question: "in what way could I present this that a *student* would find helpful?"

Changing Times

There is a growing movement in the physics community toward *accessibility* in textbooks. This was not always the case. My shelves are full of books which indulge in high-level abstraction at the reader's expense.

But readers are loosing patience with technical books which play hide-and-seek with the reader. Publishers are noticing a large, untapped market to exploit of people who don't like to feel like idiots as they read books. It turns out, pages of densely-packed equations usually *aren't* the best way to convey a broad idea to a newcomer of a discipline. Princeton University Press has launched a high-profile physics textbook series under the title *In A Nutshell*, for example, which aims for accessible but rigorous introductions to fields such as quantum field theory, string theory or Einstein's relativity.

A. Zee, a prolific author of several landmark physics texts on topics including Einstein's gravity and quantum field theory, begins his 800-page textbook on General Relativity with the following reflection on physics textbooks in general:

> Some textbook writers are simplifiers, others are what I call complicators. In defiance of Einstein's exhortation, many authors strive to make physics as complicated as possible, or so it seems to me. In the research literature, the cause of obscurity may be unintentional or intentional: either the author has not understood the issues involved completely . . ., or the author wants to impress upon the reader the profundity of his or her papers by resorting to obfuscations. But in a textbook?

I aspire to be what Prof. Zee calls a *simplifier.* The most valuable compliment I could ever aspire to in regard to this work is if a student told me they felt this book somehow *simplified* their physics experience.

Textbooks Aren't Really Meant To Be "Read"

Many textbooks are written seemingly oblivious to the fact that students won't be reading them. Odds are that students will not have time to read 350+ pages of physics textbook during a 15 week semester. At least not when every other class they have has the same expectation.

The trick is preparing a textbook which can be easily used as a quick reference but also connects ideas across a chapter in a meaningful and coherent way. If you don't have the latter piece, you're little better than a physics study guide.

There are a couple of things which this book does to try to help with quick referencing.

- Printed notes in margins highlight key points in adjacent text
- Examples are boxed and color-coded to allow students to quickly find them
- Key equations are highlighted in text
- Frequent subsections to guide students to particular discussions
- Bad examples are shown in strikethrough so that a desperate student flipping through the text for an easy answer won't write down an abomination.

The book is also type-set in LaTeX which handles and the equations, figures, examples, chapter headings and all other formatting options to give the book a uniform look.

Another fact which many writers forget is the mind doesn't normally learn by reading a minutely-detailed argument assembled from first principles. In my experience, the brain learns by gaining the broad-brush picture *first* and then filling in exceptions and nuances later on. A colleague might notice that I teaching something which is technically *wrong* or not very *precise* and then go on to be more careful about the topic in subsequent chapters. I believe in "milk before meat" though I could see a colleague claiming I've desecrated something sacred in the effort to establish the "general idea." The physics deities have not struck me dead yet for profaning the discipline, perhaps because I think I clean up after myself okay later on.

SECTION 8

Balance and Elasticity

This chapter contains two important but unrelated topics. We are going to cover the last few applications of torque (equilibrium and balance) and then discuss springs and material strength. The good news is that this is the last you will see of torque in this course.

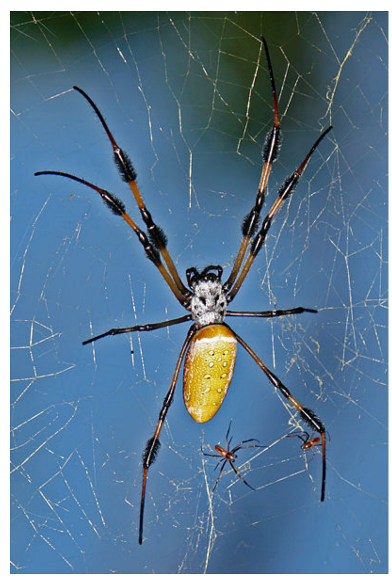


Figure 8.1: Elastic materials such as spider silk or tendons can be modeled as springs, a topic of this chapter. Credit: Tom Friedel under CC BY 3.0 US, http://www.birdphotos.com; https://commons.wikimedia.org/wiki/File:Golden-silk-spider.jpg. Wikicommons.

8.1 Torque Equilibrium

Because I introduced *center of mass* after introducing *torque* I don't feel like we ever defined torque the "right" way. Torque is what happens when a force *acts on an extended object in a way which rotates it.* Torque can happen even when an object has no pre-defined "pivot." If an object is equally free to rotate about every point, forces can create a torque about the center of mass of an object.

So the *reason* forces didn't exert torques in prior chapters is that we were subtly assuming all our masses had no spatial extent. We were effectively assuming that every mass was a point particle.

We've talked in Chapter 5 about *force equilibrium.* When forces on an object are balanced, the object didn't feel an acceleration in any direction. When *torques* are balanced on an object, the object will not feel an *angular* acceleration.

In this chapter, we're going to introduce a *new* type of equilibrium problem: *one for spatially extended objects.* Because forces on such objects aren't necessarily directed into their center of mass, some will exert a torque. But if the object is just sitting there and not rotating, **the sum of the torques on the object will equal 0.**

$$\sum \tau = \tau_1 + \tau_2 + \tau_3 + \ldots = 0 \tag{8.1}$$

When an object is static.

But we can even say something *stronger* than this. Remember that torques are defined in part by their distance from some pivot[1]. *There are plenty of extended objects which have no rotating parts.* Take a look at the hanging sign in Figure 8.2. There is no obvious place on that pole about which it should be rotating, *because the whole thing should be staying put!*

While this might appear to be a dilemma, it turns out to be a blessing. Because of this, the sum of the torques around **every point** is equal to zero. That turns out to be so important I'm going to highlight it.

For a static object, the sum of the torques about **any** point is zero.

For these static problems, *you get to choose your own pivot.* The problem works by you summing torques around your chosen pivot, setting it equal to zero and solving for a desired unknown.

That's not to say that some pivots aren't better to choose than others. There's a strategy about this that we'll show you as we start examples.

The two major types of problems which show up the most in this vein are **plank problems** and **hanging sign problems.** Both problems have the following major steps:

1. Choose a pivot point (again, there's a strategy for each problem you encounter)
2. Choose a positive rotational direction about your chosen pivot
3. Draw all the forces acting on the object
4. Sum torques and set them equal to zero (also sum forces in y direction if needed)
5. Solve for your desired unknown

We'll show you how to take care of hanging sign problems first.

Hanging Sign Problems

A pole is mounted on a wall. In Figure 8.2, a cable is attached to the far end of the pole and also mounted on the wall. The pole and cable

[1] Remember that torque has a radius r in its formula: $\tau = rF\sin\phi$.

support a sign which has some weight.

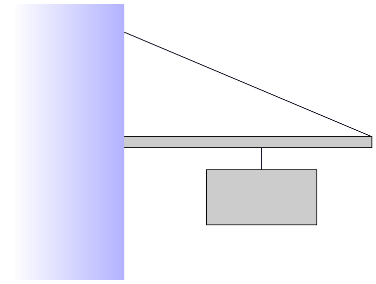
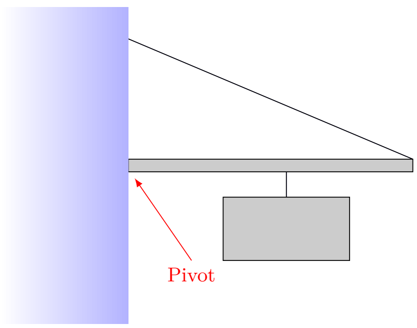


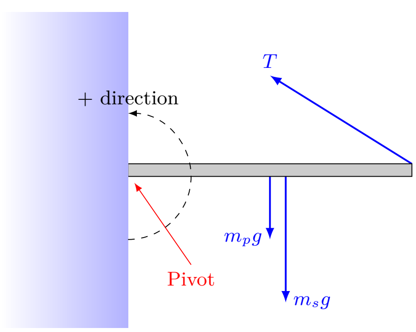
Figure 8.2: A hanging sign problem. The problem might ask *what is the tension in the cable?*

I'm going to restate the rules we just mentioned but give more detail specific for hanging signs:

1. **Choose a pivot point**
   For these problems, more often than not, **choose your pivot to be at the base of the pole.** *You can technically choose your pivot to be anywhere, but placing it at the base of the pole makes it so your don't have to worry about forces exerted on the bar there: the radius to that force will be zero!*



2. **Choose a positive rotational direction about your chosen pivot**
   I usually choose counter-clockwise as habit, but feel free to choose whichever direction you want.
3. **Draw all the forces acting on the object**
   Draw all the forces as vectors and label them. You will usually have a tension from a cable holding up the pole and at least one from some mass hanging from the pole. **Don't forget the mass of the pole itself.** The weight of the pole should be an arrow half-way along the length of the pole, *because the center of mass of an object in the absence of all other knowledge is in its center.*



4. **Sum torques and set them equal to zero (also sum forces in y direction if needed)** Make sure you remember each torque's sign. What direction would each force try to rotate the pole?

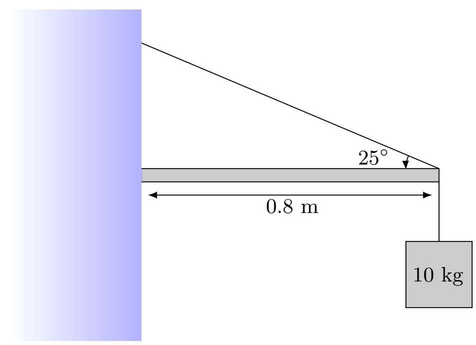
I think it is important to remember, though, that these are **just torque problems.** The only thing *different* about what we are doing is *choosing* a specific point on a object to be a pivot and *always* setting the sum of the torques equal to 0.

Let me pre-empt a possible question: *why can't we just sum forces in the x and y direction and get the answer?* On a practical level, *because these problems wouldn't give you enough information to do that.* In a hanging sign problem, there is a force provided by the wall on the beam *in addition to the unknown* which makes these problems impossible to tackle the old way. Thankfully, the objects in these problems are spatially extended. That means that all the forces acting on the object are unlikely to pass through the same point and will exert torques on the object. So *torque* becomes the way we can get at the unknown.

We are just going to jump into a series of examples. In the first, we'll find the tension in a cable attached to a pole holding up a sign. These are the vanilla problems.

Example 8.1

A 10 kg sign hangs from a 2 kg rod supported by a cable as shown. The rod is attached to a wall by a pivot. What is the **tension** of the upper cable?



Choose a pivot
We'll use the **base of the rod on the wall** as the pivot.

Choose positive rotational direction
Let's say counter-clockwise is positive. All torques that would rotate the rod clockwise about its base will henceforth be positive.

While we are at it, I'm going to redraw the figure with all the forces labeled. This appears in Figure 8.3. m_s is the **mass of the 10 kg sign**, and m_b is the **mass of the beam.**

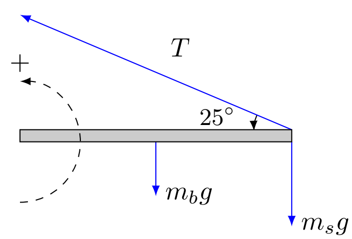


Figure 8.3: Forces are now labeled on my pole holding up the hanging sign. I've indicated the positive rotational direction I've chosen with a dashed arrow.

Don't forget the beam itself! The beam has a *weight* which acts through the beam's center of mass. *Remember that in the absence of all other knowledge, the center of mass of an object is always in its center.*

All torques add up to zero
We find the torque from each force (arrow that I have drawn) with $\tau = rF\sin\phi$.

Tension T: $+rF\sin\phi = +(0.8\text{ m})T\sin 25°$

Hanging Sign $m_s g$: $-rF\sin\phi = -(0.8\text{ m})m_s g\sin 90°$
$= -(0.8\text{ m})(10\text{ kg})(9.8\text{ m/s}^2)(1)$

Beam's mass $m_s g$: $-rF\sin\phi = -(0.4\text{ m})m_b g \sin 90^\circ$
$$= -(0.4\text{ m})(2\text{ kg})(9.8\text{ m/s}^2)(1)$$

All of these will add to zero:

$$+ \quad (0.8\text{ m})T\sin 25^\circ \quad -(0.8\text{ m})(10\text{ kg})(9.8\text{ m/s}^2)(1) \quad - \quad (0.4\text{ m})(2\text{ kg})(9.8\text{ m/s}^2)(1) = \underbrace{0}_{static}$$

The tension T can be isolated and solved for. Add over the 2 terms not containing T and divide both sides by $(0.8\text{ m})\sin 25^\circ$. We get

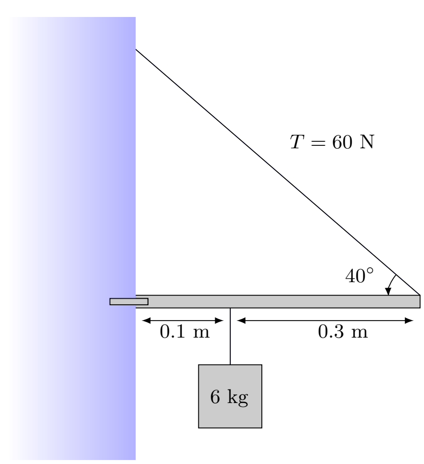
$$T = \frac{(0.8\text{ m})(10\text{ kg})(9.8\text{ m/s}^2) + (0.4\text{ m})(2\text{ kg})(9.8\text{ m/s}^2)}{(0.8\text{ m})\sin 25^\circ} \boxed{= 255 \quad \text{N.}}$$

There's another variant of the problem where you are asked for just the needed torque to keep a system from moving. Sometimes they'll ask how much torque an odd object must exert to do this *when we have been given no information on how this object works.* This is okay, it turns out. *we actually don't need to know that.* All they are asking about is torque. And this is an easy quantity to calculate.

Let's do another example.

Example 8.2

What **torque** must the brace at the base of the 1.5 kg bar exert on the bar to keep it stationary?



If the bar is ***stationary****, the sum of the torques on the bar is 0. Let's find the* ***net torque*** *on the bar with all the labeled forces. The brace must be providing* ***exactly opposite of that.***

Choose a pivot
Let's again choose the base of bar as the pivot.

Choose positive rotational direction
Let's say counter-clockwise is positive. As with the other examples

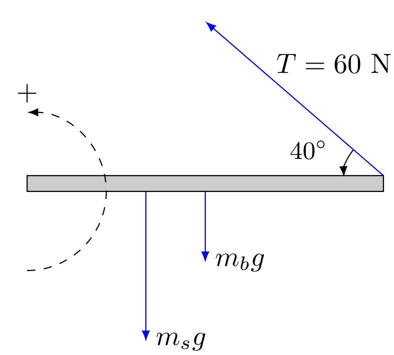


Figure 8.4: Illustration for Example 7.2 with forces and positive direction labeled.

I'm going to redraw the figure with all the forces labeled. This appears in Figure 8.4. m_s is the **mass of the 10 kg sign**, and m_b is the **mass of the beam.**

All torques add up to zero
We find the torque from each force (arrow that I have drawn) with $\tau = rF\sin\phi$.

Tension T: $+(0.4\text{ m})(60\text{ N})\sin 40^\circ$

Weight of beam $m_b g$: $-(0.2\text{ m})m_b g\sin 90^\circ$
$= -(0.2\text{ m})(1.5\text{ kg})(9.8\text{ m/s}^2)$

Hanging sign $m_s g$: $-(0.1\text{ m})m_s g\sin 90^\circ$
$= -(0.1\text{ m})(6\text{ kg})(9.8\text{ m/s}^2)$

Torque from brace τ_1

Again, *all* of these torques add up to zero:

$$\begin{aligned} +(0.4\text{ m})(60\text{ N})\sin 40^\circ & \quad -(0.2\text{ m})(1.5\text{ kg})(9.8\text{ m/s}^2) \\ - & \quad (0.1\text{ m})(6\text{ kg})(9.8\text{ m/s}^2) + \tau_1 = \underbrace{0}_{\text{static}} \end{aligned}$$

This can be solved for τ_1 by subtracting $(0.4\text{ m})(60\text{ N})\sin 40^\circ$ and adding $(0.2\text{ m})(1.5\text{ kg})(9.8\text{ m/s}^2)$ and $(0.1\text{ m})(6\text{ kg})(9.8\text{ m/s}^2)$ to both sides. We get

$$\begin{aligned} \tau_1 &= -(0.4\text{ m})(60\text{ N})\sin 40^\circ + (0.2\text{ m})(1.5\text{ kg})(9.8\text{ m/s}^2) \\ & \quad +(0.1\text{ m})(6\text{ kg})(9.8\text{ m/s}^2) \boxed{= 6.6\ \text{ N m.}} \end{aligned}$$

Hanging sign problems have a consistent rhythm to them. These two examples are pretty representative of what you will see.

These problems, more than Chapter 5 problems, have a **consistent rhythm.** If you can get the rhythm down, you can pretty well crack any problem which comes your way.

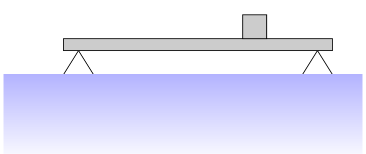


Figure 8.5: Plank problems involve a plank laying across some objects. A question might ask *what is the upward force through the left fulcrum?*

Plank Problems

In these problems a plank (a spatially extended object) is stretched over some objects. Often the plank rests on a **fulcrum** which is a fancy word for a **triangle-shaped object** which, in the absence of the other objects, the plank could tip over.

Again, this might not strike you as a *rotational* problem, so *why are we using torques again?* The plank is spatially *large* which means that forces acting on the board *have a strong change of not being directed through its center of mass.* So most forces will exert a torque on the object.

And because *nothing is moving* in this situation, the sum of those torques had better be equal to zero.

I'm going to again restate the rules for solving torque problems, but this time be specific to plank problems.

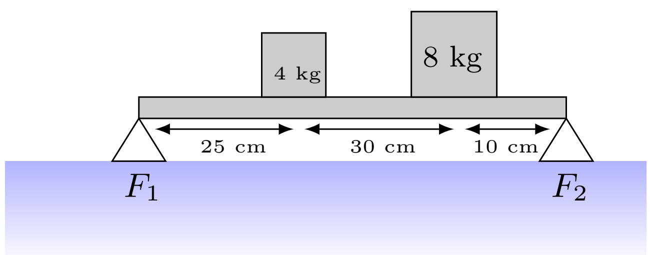
1. **Choose a pivot point**
   For statics problems, choose the fulcrum you are **not** interested in at your pivot. Choosing a pivot on the location where a force is acting eliminates the force from your torque equation.
2. **Choose a positive rotational direction about your chosen pivot**

3. **Draw all the forces acting on the object**
   Draw all the forces as vectors and label them. All fulcrums exert an upwards force no the board and all masses on top exert a downwards force. **Don't forget the mass of the pole itself.** The weight of the plank should act at half-way along the length of the plank,
4. **Sum torques and set them equal to zero (also sum forces in y direction if needed)** Make sure you remember each torque's sign. What direction would each force try to rotate the plank about your chosen pivot?

We'll show you how these fit together in an example.

Example 8.3

What **force** does fulcrum 1 (F_1) exert on the 3.0 kg plank?



Why can't I just sum forces in the y direction to get the answer? Because there are **2** unknowns: F_1 and F_2 are both not given. One equation and 2 unknowns does not a happy physics student make. Let's sum torques instead. (Plus there are **fulcrums** (triangles) in this problem which gives it away that I need to use **torques** to solve the problem.)

Choose a pivot
We want to choose a fulcrum as the pivot. Choose the fulcrum which *isn't* the unknown. We'll choose fulcrum 2.

Choose positive rotational direction
Let's choose counter-clockwise as positive. I'm going to redraw the plank with all of the torques acting on it in Figure 8.6. **The curved arrow indicates my chosen positive direction.**

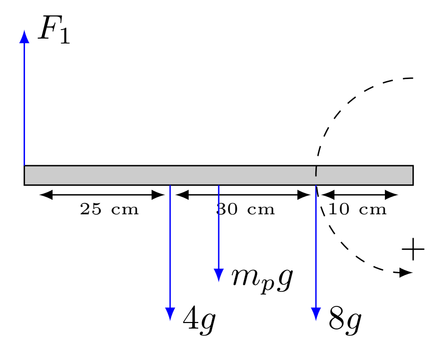


Figure 8.6: Forces and chosen positive direction for the plank problem in Example 7.3

Note that I have included the weight of the plank m_pg. The plank's weight acts through the plank's center-of-mass: the middle of the plank. The plank is 65 cm long, so it will act at 32.5 cm from our pivot.

All torques add up to zero
We find the torque from each force (arrow that I have drawn) with $\tau = rF\sin\phi$. *Because all forces are perpendicular to the plank, I'm going to simplify this to* $\tau = rF$. This is okay because $\sin 90^\circ = 1$.

Fulcrum 1 F_1: $-(0.65\text{ m})F_1$

4 kg box $4g$: $+(.40\text{ m})(4g) = +(.40\text{ m})(4\text{ kg})(9.8\text{ m/s}^2)$

8 kg box $8g$: $+(.10\text{ m})(8g) = +(.10\text{ m})(8\text{ kg})(9.8\text{ m/s}^2)$

Plank m_pg: $+(.325\text{ m})(m_pg) = +(.325\text{ m})(3\text{ kg})(9.8\text{ m/s}^2)$

All of these will add to zero:

$$\begin{aligned} & -(0.65\text{ m})F_1 + (.40\text{ m})(4\text{ kg})(9.8\text{ m/s}^2) \\ + \quad & (.10\text{ m})(8\text{ kg})(9.8\text{ m/s}^2) + (.325\text{ m})(3\text{ kg})(9.8\text{ m/s}^2) = \underbrace{0}_{\text{static}} \end{aligned}$$

Subtract over all terms which do not include F_1 and then divide both sides by -(0.65 m) to find F_1:

$$\begin{aligned} -(0.65\text{ m})F_1 \quad = \quad & -(.40\text{ m})(4\text{ kg})(9.8\text{ m/s}^2) \\ & -(.10\text{ m})(8\text{ kg})(9.8\text{ m/s}^2) \\ & -(.325\text{ m})(3\text{ kg})(9.8\text{ m/s}^2) \end{aligned}$$

We now divide both sides by -0.65 m and get

$$F_1 = \boxed{50.88\text{ N.}}$$

8.2 Balance

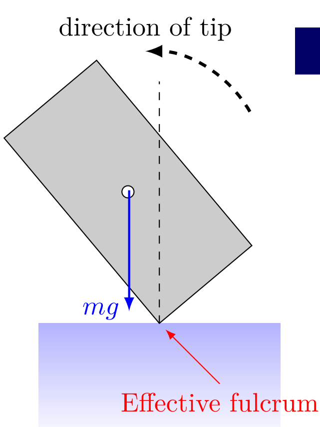


Figure 8.7: When an object tips, one point of the object remains on the ground: an effective fulcrum. Whatever side of the fulcrum the center of mass falls will be the side the object tips to.

If you were to step out onto a sheet of ice unexpectedly, how would you stand to avoid falling over? Instinctively, you would put your feet far apart and crouch lower. Your body is exploiting a key point about balance: if your center of mass is directly over a (now much larger) base of support, *you won't fall over.*

Just because an object *tips* doesn't mean that it will *tip over*. You push on a dresser and it may fall back from where it was standing. But push it *too far* and the story is different. What makes the difference? *Whether or not the center of mass of the dresser remains over its base of support.*

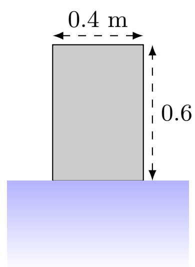
Said another way, as anything tips, a single point of the object remains touching the ground. This is like an **effective fulcrum** (see Figure 8.7). Which ever side of the fulcrum the gravitational force from the center of mass of the object falls is how the object will tip.

What this means is we can now predict the tipping angle for an object. This is summed up in the following idea:

> The critical tipping angle θ of an object happens when the center of mass is above the effective fulcrum.

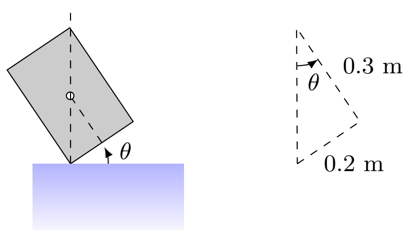
Example 8.4

Calculate the tipping angle for the object below.



The object will be on the verge of tipping when its center of mass (presumably in the middle of the object because I have no reason to believe otherwise) is over the effective fulcrum.

I'm going to draw the object again, but tipping so that its center of mass is just over the fulcrum.



Once I do this, you can see that there's a right triangle which forms there. I've drawn a blow up of it off to the right. Because the center of mass is in the middle of the object, the sides of this triangle are half the length and width of the box.

Notice also that the angle between the vertical and the tilted vertical direction is *also* θ. This reason for this is similar to the argument about the inclination angle of a slope is the same as the angle between the direction of gravity and the negative y axis.

You can now use $\tan\theta$ =opposite/adjacent to solve for θ:

$$\theta = \tan^{-1}\left(\frac{0.2\text{ m}}{0.3\text{ m}}\right) = \boxed{33.7^\circ.}$$

Most problems about tipping angle can be solved this way. Draw the object in a configuration where the center of mass of the object is directly above the object's sole contact point (or effective fulcrum), construct a right triangle, and solve for the angle of interest.

Figure 8.8: If the center-of-mass of an object is above the base of its support, it will balance. This bottle and board, constructed by Prof. Rhett Zollinger, demonstrates this dramatically. The combined center of mass for this glass bottle and wood board is position *directly* over the board's small edge, allowing it to balance.

What The Critical Angle Means

The critical angle is a rough measure of what we call *stability.* The larger the critical angle, the more stable the object is against tipping over.

Objects with relatively high centers of masses are less stable because a smaller tip will place the object's center of mass outside the base of support. This agrees with intuition: when an object is *top heavy* it is easier to *topple.*

Cars are rated by their base-to-height ratio (called a Static Stability Factor or SSF) and objects with low base-to-height ratios are labeled has having a higher tipping risk. "Taller" vehicles need to take corners slowly because they are in greater danger of rolling.

Teeter Totters

Physics problems involving balance can frequently involve *multiple objects.* A handy example is "teeter totter" problems. A board is placed on a fulcrum with a mass on one side and we are asked where to place second mass or we are asked *how much* that mass should be. There are two ways to go about these problems.

- **We can sum torques**, taking the location of the fulcrum of the teeter totter as the pivot. We set the net torque to 0 which indicates rotational equilibrium and solve for the desired unknown.

  $$\sum \tau = r_1 F_1 + r_2 F_2 + r_3 F_3 + \ldots = 0$$

  The pivot will be the location of the fulcrum

  Notice that I've left off the $\sin\phi$ on each torque term (it should be e.g. $r_1 F_1 \sin\phi$). I've done this because the torques in these problems are *weights* which often act a right angles with respect to the radial.

- **Or we can use the center of mass formula** and require that the center of mass of the system be directly over the fulcrum.

  $$x_{\text{com}} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

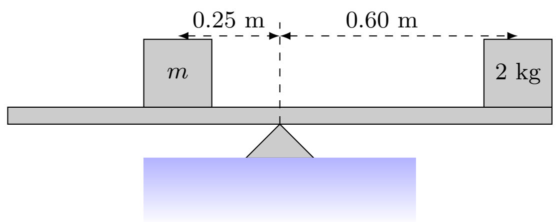
  The center of mass should be at the location of the pivot.

If you look closely at the formulas, they are actually saying the exact same thing. $r_1 F_1$ in one formula parallels $x_1 m_1$ in the second formula. Nevertheless, you'll probably develop a preference for which formula you end up using to solve these problems.

I going to present a problem where we solve it each way over two examples. You can pick the method which works well for you.

Example 8.5

Two masses sit on a teeter totter as shown. What is m the mass of the leftmost block which balances the system? Assume the board is massless.



We are going to **sum torques** to solve this.

I've **labeled forces** and **chosen a positive rotational direction** in Figure 8.9. Remember that the board is *massless*, so I don't include a weight for it.

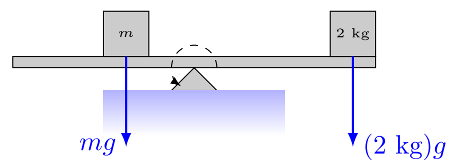


Figure 8.9: Positive direction and forces of the two blocks on the balancing plank.

I now sum torques

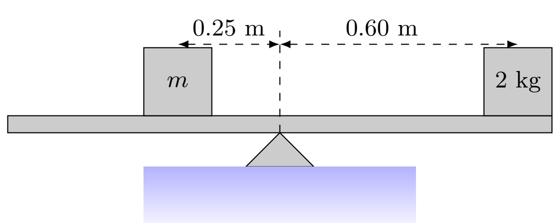
$$\sum \tau = (0.25\ \text{m})m(9.8\ \text{m/s}^2) - (0.60\ \text{m})(2\ \text{kg})(9.8\ \text{m/s}^2) = \underbrace{0}_{\substack{\text{system}\\ \text{balances}\\ \text{(literally!)}}}$$

All we need to do is solve for m. Adding $(0.60\ \text{m})(2\ \text{kg})(9.8\ \text{m/s}^2)$ term and dividing by $(0.25\ \text{m})(9.8\ \text{m/s}^2)$ gives

$$m = \frac{(0.60\ \text{m})(2\ \text{kg})(9.8\ \text{m/s}^2)}{(0.25\ \text{m})(9.8\ \text{m/s}^2)} = \boxed{4.8\ \text{kg}}$$

Example 8.6

Two masses sit on a teeter totter as shown. What is m the mass of the leftmost block which balances the system? Assume the board is massless.



We are going to **use the center of mass formula** to solve this.

I'm going to choose my origin to be the center of the fulcrum. That means that the block of mass m is located at *negative* 0.25

m (remember that signs matter in the center of mass formula) and the 2 kg block is located at +0.60 m because it is to the right of the origin. I also know that that the center of mass of the system, if it is balanced, is located directly over the fulcrum, $x_{\text{com}} = 0$ m. Plugging this all into the center of mass formula gives

$$(0 \text{ m}) = \frac{(-0.25 \text{ m})m + (0.05 \text{ m})(2 \text{ kg})}{m + 2 \text{ kg}}$$

Multiplying both sides by $m + 2$ kg gives

$$(0 \text{ m}) = (-0.25 \text{ m})m + (0.60 \text{ m})(2 \text{ kg})$$

so, solving for m gives

$$m = \frac{-(0.60 \text{ m})(2 \text{ kg})}{-0.25 \text{ m}} = \boxed{4.8 \text{ kg}}$$

Either strategy gets you the right answer, so you feel free to take your pick as to which method is easier.

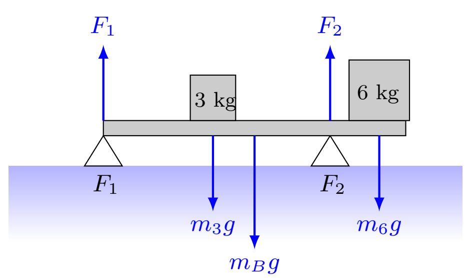
Walk The Plank

What is the farthest an object can be placed on a board hanging over a ledge before the plank and box tip over the edge? This is *another* question which can be answered with either torques or a center of mass argument.

These are just glorified **torque problems** with one twist[2]. These problems ask either how *large* on object can be or how *far out* past an edge an object can be placed before the system tips.

But what happens when something tips?

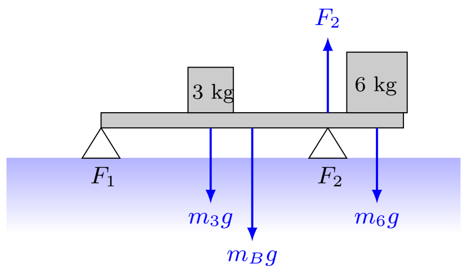
Normally, the torques acting on a board might look like this:



When a board is *just about to tip over some point,* that point provides the *only upward force* on the board.

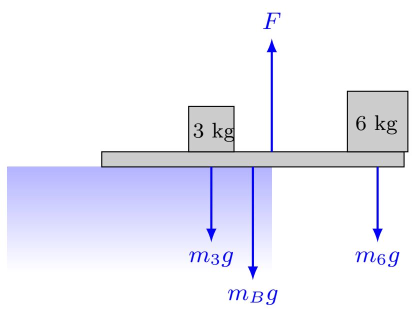
But say that the board is about to *tip* over fulcrum 2 (F_2). At the very instant the board starts to tip. The forces on the board actually look like this:

[2] I've waited a full chapter for that pun.



Which is to say that the left-most fulcrum *no longer supplies an upward force.* Why? When the board is *just barely about to tip*, it is *balancing* on the fulcrum about which it will tip over. That fulcrum provides the *only* upwards force at that instant.

The same thing takes place with boards on tables, by the way. *Normally,* the table would exert a normal force along the length of the board. *But when the board is about to tip*, the board momentarily balances on *the edge* of the table, *and the table's edge provides the only upwards force.*



Why does any of this matter? This discussion boils down to two things:

When a board is about to tip

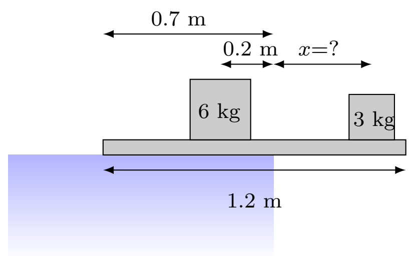
1. The only upward forces is provided by the effective fulcrum.
2. The location of your chosen pivot when you sum torques should be the effective fulcrum (a.k.a. tipping point).

What this means is that when you have a problem which asks about conditions when a system is going to tip, you need to identify the tipping pivot of the system. The problem only becomes solvable when you choose this fulcrum as your pivot because forces acting through pivots don't appear in torque summations: you don't want to deal with the unknown upwards force provided by the fulcrum as your solving for your unknown.

Once last piece of counsel: when you are doing these tipping problems, *watch carefully for how the problem defines the length they are looking for.* If the problem asks for a distance between the effective fulcrum and the mass, solving for it is straightforward. But sometimes this is not the distance they ask for. I'm going to carry out two examples of an identical problem but in which each example asks for the distance in different ways. See if you can spot the difference in my approach.

Example 8.7

Two blocks rest on a 2.0 kg board. What is the maximum distance x that a block can be placed before the board tips over?



We are going to sum torques about the edge of the table:

$$\sum \tau = -(6\text{kg})(9.8\text{ m/s}^2)(0.2\text{ m}) - (2\text{kg})(9.8\text{ m/s}^2)(0.1\text{ m}) + (3\text{ kg})(9.8\text{ m/s}^2)x = 0$$

Notice that there are only "downward" forces in this equation (see Figure 8.10). An upward force *is* provided by the edge of the table, but because that also happens to be where we have chosen to put our pivot, the radius to that force is 0 m and so it doesn't show up in our equation. x is now easy to solve for:

$$x = \frac{(6\text{kg})(9.8\text{ m/s}^2)(0.2\text{ m}) + (2\text{kg})(9.8\text{ m/s}^2)(0.1\text{ m})}{(3\text{ kg})(9.8\text{ m/s}^2)} = \boxed{0.467\text{ m.}}$$

If I place the board any farther from this, the board and blocks will tip over the edge of the table.

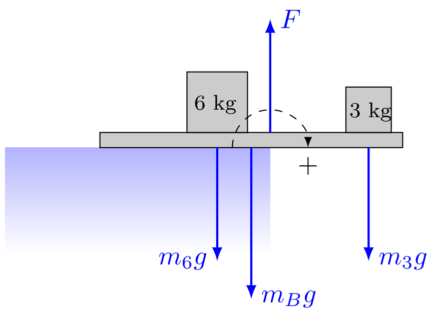
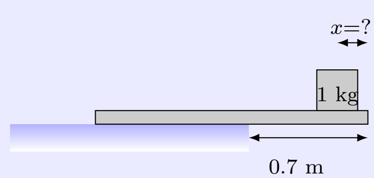


Figure 8.10: Labeled forces and positive direction for Example 8.7. Our choice of pivot is at the edge of the table. Notice that the only upward force at the moment of tipping is at location of our pivot (tipping point at edge of table) and so it doesn't show up in our torque equation.

In the above example, the distance they asked for was the same as the distance between the effective fulcrum and the box. *That's the same as asking for a radius of the box to the pivot* which is why I just put x in for r. But sometimes you *have to be on your toes.*

If, for instance, they ask for the minimum distance from the edge a mass can be without causing things to tip over, you handle this by subtracting x from the length of board hanging off the table. It is *this quantity* which you insert for r.

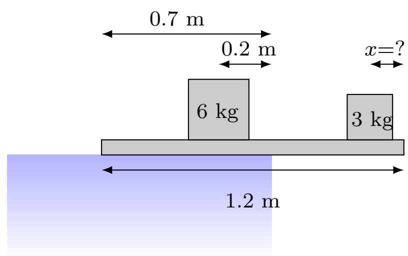
Quick example: What is the radius to the box?



The distance to the box from the edge of the table (effective fulcrum) would be $(0.7\text{ m} - x)$. This would be what I would use for r when adding the torque for the 1 kg box.

Example 8.8

Two blocks rest on a 2.0 kg board. What is the **minimum** distance x that a block can be placed before the board tips over?



I *might* be tempted to sum torques this way:

$$\begin{aligned}\sum \tau &= -(6\text{kg})(9.8\text{ m/s}^2)(0.2\text{ m}) - (2\text{kg})(9.8\text{ m/s}^2)(0.1\text{ m}) \\ &\quad + \cancel{(3\text{ kg})(9.8\text{ m/s}^2)}x = 0\end{aligned}$$

But this is **wrong** because the torque equation needs the distance from the block to the pivot, *not the distance from the block to the edge of the plank*.

The amount of board hanging over the edge of the table is 0.5 m (it's a 1.2 m plank and 0.7 m of it is on the table). Is the *distance* between the 3 kg block and the edge of the table would be 0.5 m $-x$. So our equation *should* look like

$$\begin{aligned}\sum \tau &= -(6\text{kg})(9.8\text{ m/s}^2)(0.2\text{ m}) - (2\text{kg})(9.8\text{ m/s}^2)(0.1\text{ m}) \\ &\quad + (3\text{ kg})(9.8\text{ m/s}^2)(0.5\text{ m} - x) = 0\end{aligned}$$

This equation will take a little bit of work to solve.

$$(0.5\text{ m} - x) = \frac{(6\text{kg})(9.8\text{ m/s}^2)(0.2\text{ m}) + (2\text{kg})(9.8\text{ m/s}^2)(0.1\text{ m})}{(3\text{ kg})(9.8\text{ m/s}^2)}$$

I'm going to simplify the right-hand-side:

$$(0.5\text{ m} - x) = 0.467\text{ m}$$

which simplifies to

$$x = 0.5\text{ m} - 0.467\text{ m} = \boxed{0.333\text{ m.}}$$

I will also point out that if you get a problem which asks for an awkward distance like this, you *can* solve it like Example 8.7 above and *then* take your answer and subtract it from the length of the plank hanging over the edge of the table. I would accept either way. Just know that when the requested distance is given from the edge of the plank, there's a little more work involved than when it is requested from the pivot.

Beware of problems which ask for distances which are **NOT** the distance between a mass and the pivot.

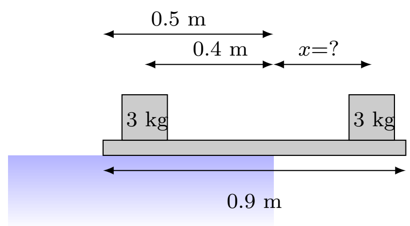
There is a yet *another way* to solve these problems which students find conceptually easy as well. You can use the center of mass formula and set the center of mass of the blocks and boards to be the edge of the

You can also use the center of mass formula to solve "tipping" problems table or the location of the tipping fulcrum.

The physics principle that you're using here is that something is balanced *so long as its center of mass is over the base of support.* Thus the *maximum* distance some mass can be placed from the edge of a table happens when the center of mass for the *whole* system is right at the edge of the table. If you place the block at any greater distance, the center of mass for the whole system will move beyond the table and everything will tip over the edge.

Example 8.9

Two blocks rest on a 2.0 kg board. What is the maximum distance x that a block can be placed before the board tips over?



We're going to show you how to use the center of mass formula to tackle this. The *maximum allowable center of mass for the system is the edge of the table.* Any farther than this, and everything goes over the edge. I'm going to set my origin to be the edge of the table.

$$(0 \text{ m}) = \frac{(-0.4 \text{ m})(3 \text{ kg}) + (-0.05 \text{ m})(2 \text{ kg}) + (3 \text{ kg})x}{3 \text{ kg} + 2 \text{ kg} + 3 \text{ kg}}$$

Multiplying both sides by the denominator gives

$$(0 \text{ m}) = (-0.4 \text{ m})(3 \text{ kg}) + (-0.05 \text{ m})(2 \text{ kg}) + (3 \text{ kg})x$$

We solve for x to get

$$x = \frac{(0.4 \text{ m})(3 \text{ kg}) + (0.05 \text{ m})(2 \text{ kg})}{3 \text{ kg}} = \boxed{0.433 \text{ m.}}$$

For whatever reason, some students really like this way. You compare this way with that of the previous two examples and take your pick.

I've been talking in terms of planks, but the principles in this section can be applied to other things too.

When you purchase large dressers, most now come with wall anchors. Some dressers only allow one drawer open at a time. These are both motivated by principles of tipping. Having lots of heavy drawers filled with clothes open at once is like moving a block out on a plank: all those open drawers may move the dresser's center of mass beyond the base of its support and cause it to tip. The same thing could happen if a toddler attempted to *climb* a heavy dresser, a horror scenario every parent dreads. It turns out those wall anchors come with those dressers for a reason.

8.3 Springs

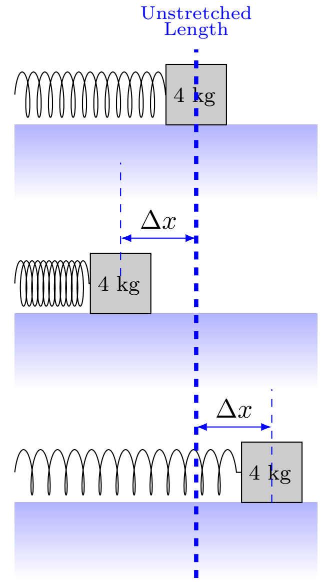


Figure 8.11: Δx is the amount of compression or stretch of a spring from its un-stretched length

You can breathe a bit easier now. Torques are gone.

I'm going to take the time in this chapter to talk about *springs*. This may seem like an *odd* thing to talk about, especially because few of us aspire to be toy makers. But there are a *lot* of things in nature which are elastic which we model *as if they were springs*. Things as different as tendons and the Higgs boson's scalar field obey the very basic spring physics that we learn in this section.

Some forces get weaker as you move farther from the source of the force. Gravity is an example: the more distant I am from Earth, the weaker its gravitational force on me becomes. But some forces *get stronger* as the source of the force gets farther away. The spring is the archtypical example: the more I stretch a spring, the stronger a force it exerts on me.

Every spring has a **natural** or **unstretched length**. When the spring is at this length, it exerts no force. But when the spring is *different* from this length, either stretched or compressed, the spring exerts force. We'll call the **stretched or compressed length** of the spring Δx (see Figure 8.11).

The force the spring exerts F_s is

Change In Length

$$\Delta x$$

This is how much spring's length differs from its natural length. It is *not* the total length of the spring.

Hooke's Law

$$F_s = k\Delta x \tag{8.2}$$

in the direction opposite of the stretch or compression

The constant k is the **spring constant**. It has units of Newtons per meter, N/m. It tells you how stiff the spring is: the higher the k value, the stiffer the spring.

Spring Constant

$$k$$

The spring constant tells us how "stiff" the spring is that we are stretching. A bigger value of k means that the spring is harder to stretch or compress. It has units of **N/m**.

This "law" governs how springs stretch and is called "Hooke's Law." Some books will write this equation as $\vec{F}_s = -k\vec{\Delta x}$, the negative referencing the "opposite direction" the spring force has compared to Δx. I find the negative a bit pedantic and confusing: anyone with common sense knows which way the spring force is felt. You might also see other textbooks write this as $F_s = kx$. I avoid this also because you can confuse x with the length of the spring instead of the **difference** between the spring's length and its natural length.

I'll show you what I mean in the example below.

Quick example: A spring ($k = 120$ N/m) is stretched 14 cm beyond its natural length. What is the magnitude of the force exerted by the spring?

We use equation (8.2) and get

$$F_s = (120 \text{ N/m})(0.14m) = \boxed{16.8 \text{ N.}}$$

Notice we had to convert to m.

This next example tries to distinguish *natural* length and *stretched* length of the spring.

Quick example: A spring ($k = 130$ N/m) has a natural length of 8 cm. It is stretched to a length of 10 cm. What is the magnitude of the force exerted by the spring?

The Δx in (8.2) is *not the total length of the spring.* It is just the *stretched length.* So our calculation of the force looks like

$$F_s = (130 \text{ N/m})(0.1 \text{ m} - 0.08 \text{ m}) = \boxed{26 \text{ N.}}$$

I had to convert the lengths to meters. Notice that I *subtracted off* the natural length of the spring. The natural length doesn't count in this equation.

Spring Force Directly Proportional To Δx

The spring force is directly proportional to Δx. This means that the if the change in length of spring *doubles* the force from the spring will also *double.*

You can encounter problems which play with this direct proportionality game.

Quick example: A spring is stretched 14 cm beyond its natural length and exerts a force of 4 N. The spring is then relaxed so that it is stretched only 7 cm. What forces does the spring exert?

Wait a minute! They didn't give us a spring constant! But it doesn't matter. Because the formula $F_s = k\Delta x$ means that F_s is directly proportional to Δx. If the stretched length in *halved* then so is the force. The answer is $\boxed{2 \text{ N.}}$

There are also these games where the spring constant is not initially known but must be calculated to complete the problem. Some clever people use a proportionality argument. But I usually just calculate the spring constant from the data and use it to answer the question later posed by the problem.

Example 8.10

A spring is stretched 8 cm and exerts a force of 40 N. How much force would it give if it were stretched 20 cm?

Find the spring constant k
We use $F_s = k\Delta x$:

$$(40\ \mathrm{N}) = k(0.08\ \mathrm{m}) \qquad \Rightarrow k = \frac{40\ \mathrm{N}}{0.08\ \mathrm{m}} = 500\ \mathrm{N/m}$$

Then use the spring constant to calculate new force
Again use $F_s = k\Delta x$ to get

$$F_s = (500\ \mathrm{N/m})(0.2\ \mathrm{m}) = \boxed{100\ \mathrm{N.}}$$

Theoretical Aside: Hooke's Law Applies Beyond Just Springs

Do all real springs behave like equation (8.2)*?* We just introduced $F_s = k\Delta x$ in the context of springs, so it might surprise you that the answer is *no*. There are plenty of springs that do ... to a point. Stretch the spring *too far*, however, and the spring will quit supplying a restoring force. We'll talk about what is happening to the spring as it stretches into this extreme limit in the next section.

Equally surprising is that most solid materials *obey* Hooke's Law ... for a very small amount of stretching. If you pull on both sides of a solid brick, *microscopically* the brick stretches like a very, very stiff spring. Let go and *microscopically* it will oscillate about its relaxed length. The normal force is itself a spring phenomenon. How does the table "know" to push up a 2 kg book *exactly* (2 kg)(9.8 m/s^2)? The answer is that it *doesn't* but that the table is like a "mattress" of microscopic springs. As the book sinks microscopically into the table, the springs in the table supply more and more force until the force from the springs of the table (molecular bonds) and the weight of the book exactly match. Then the book quits sinking, being supported by a network of stretched/compressed molecular bonds beneath it.

Figure 8.12: The Higgs field, the essence which fundamental particles interact with to get their masses, and the inflaton field, the particle which spurred the acceleration of the universe, are modeled in part with Hooke's Law. Credit: ESO.

The point is when we discuss Hooke's law, we are discussing much more than springs. Virtually everything which is not a liquid or a gas can be modeled with it, if only in a limited regime.

But the springs don't stop there. Hooke's law is used to study some of the fundamental underpinnings of reality. The scalar field of the Higgs boson, the particle which gives fundamental particles their mass is modeled with Hooke' Law. As you pore over quantum field theory textbooks, you'd naively figure the model would be more sophisticated than just a *spring*, but Hooke's Law appears, as clear as day. The strong nuclear force, the force which binds quarks and the nucleus of the atom together, is *also* a $F_s = k\Delta x$-type force which gets stronger as nucleons move farther apart.

I remember the college lecture at SUU when I first learned about springs. I rolled my eyes and thought *what interest are mechanical springs to an aspiring astrophysicist?* Every solid object, from an elastic band to the crust of a neutron star has a spring-like regime. And where there was no physical elasticity, a spring became a *metaphor* for a force whose strength increased with distance. In these 6 years since I entered grad school, I have not been able to get away from them.

And trust me, I've tried.

Springs and Newton's 2nd Law

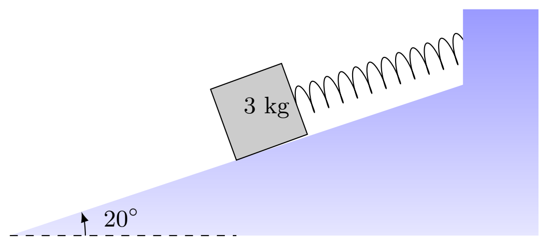
Because springs can supply a force, you can imagine that they sometimes appear in $F = ma$-type problems.

In the homework, I've been you a comparatively easy problem with a spring. I'm going to risk a more tricky problem as an example.

The key to remember is that a spring is just a *force* and you deal with it just like other forces.

Example 8.11

A block rests on a frictionless slope. How much is the spring ($k = 50$ N/m) stretched beyond its relaxed length?



We draw our free body diagram, complete with tilted axes (Figure 8.13). I'm going to sum forces in the tilted x direction:

$$\sum F_x = F_s + mg\cos 250^\circ = 0$$

Remember that the angle between mg and the negative, titled y axis is equal to the angle of the slope. The question is asking for Δx, so we plug in $F_s = k\Delta x$ with $k = 50$ N/m and get

$$(50 \text{ N/m})\Delta x + (3 \text{ kg})(9.8 \text{ m/s}^2)\cos 250^\circ = 0$$

Solving for Δx gives

$$\Delta x = \frac{-(3 \text{ kg})(9.8 \text{ m/s}^2)\cos 250^\circ}{50 \text{ N/m}} = \boxed{0.2 \text{ m.}}$$

The spring stretches 20 cm beyond its relaxed length.

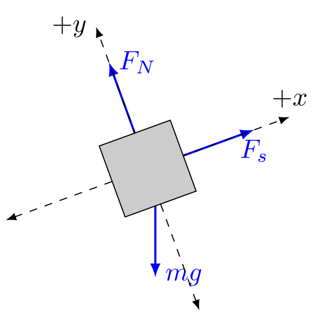


Figure 8.13: Free body diagram for a block attached to a spring on a frictionless slope.

You can see I dealt with the spring force a bit like I do friction or weight. I first put in F_s when I'm summing forces in the x and y directions and *then* substitute in $k\Delta x$ to solve for my desired unknown. Like mg and friction, the spring force is just another force which has a *formula*. You end up substituting in the formula for that force to solve the problem. Not hard to deal with at all.

8.4 Strength, Stress and Strain

Tension is what happens when you take an object by opposite ends on pull in opposite directions. Ropes come under *tension* when they are tight, or when they are being pulled by both ends.

Tension can happen with other objects too. A steal beam, pulled from both ends in opposite directions, is said to be *under tension.* Tension is opposite of *compression* where an object feels forces from both sides pressing *inward.*

Things *stretch* under tension, if only slightly. *How much* and object stretches is dictated by how *elastic* the substance is. We report the lack of elasticity of an object with the **elastic modulus** a.k.a. the **Young's modulus** E. For example, Titanium ($E = 110.3$ GPa) would have an elastic modulus much *larger* than, say, a rubber band ($E = 0.1$ Gpa).

Elastic Modulus

$$E$$

A measure of the lack of elasticity of a material (bigger numbers mean the substance is more stiff). It is usually reported in units of giga pascals **GPa**.

How much something stretches *or compresses* because of a force not only depends on the material, but *how much* material is present. A 0.2 cm-diameter steel beam will stretch more under a force than a 1 cm-diameter steel beam. So the change in length ΔL is expected to be function of the area of the object perpendicular to the stretch or compression. We call this area (A) the *cross-sectional area* of the object.

Quick example: A 1-m diameter circular cement pillar supports a large tell beam. What is the pillar's cross sectional area?

The cross sectional area would be the area of a circle with a 1-m diameter:

$$A = \pi(0.5 \text{ m})^2 = \boxed{0.785 \text{ m}^2}$$

If an object is under tension from some force F which has an original length of L, then the following relation is true:

$$\frac{\overbrace{F}^{\text{force}}}{\underbrace{A}_{\text{cross-sectional area}}} = \underbrace{E}_{\text{elastic modulus}} \frac{\overbrace{\Delta L}^{\text{change in length}}}{\underbrace{L}_{\text{original length}}} \tag{8.3}$$

In equation (8.3), E should be given in Pa, not GPa. You should almost always have a "something $\times 10^9$" in for E.

This formula works for both **compression** and **tension**. To work, **everything needs to be in SI units** which means that E should be given in Pa not GPa.

Quick example: A 1.5 mm-diameter copper wire ($E = 117$ GPa), originally 1.2 m in length, is stretched with a force of 5 kN. What is the new length of the wire?

We use equation (8.3), but first I'm going to get somethings ready. The cross-sectional area A is the area of a circle with a diameter of 1.5 mm, or a radius of 7.5×10^{-4} m. So $A = \pi r^2 = \pi(7.5 \times 10^{-4}\text{ m})^2$. The problem gives me the elastic modulus E in terms of GPa, *but we need to use just Pa for the equation.* A "giga" is 10^9 so we really have $E = 177 \times 10^9$ Pa. Now we plug all this into equation (8.3) and get:

$$\frac{5000\text{ N}}{\pi(7.5 \times 10^{-4}\text{ m})^2} = (117 \times 10^9\text{ Pa})\frac{\Delta L}{1.2\text{ m}}$$

So ΔL is then

$$\frac{5000\text{ N}}{\pi(7.5 \times 10^{-4}\text{ m})^2}\left(\frac{1.2\text{ m}}{117 \times 10^9\text{ Pa}}\right) = \boxed{0.029\text{ m.}}$$

Quick example: A square 0.5 m × 0.5 m diamond[a] ($E = 1100$ GPa) pillar supports a 6×10^6 kg roof in a building. If the pillar is originally 10.0 m tall, by how much is it compressed?

The 6×10^6 kg is a *mass*, not a *force*. To turn this into a weight, we need to multiply it by g. So $F = (6 \times 10^6\text{ kg})(9.8\text{ m/s}^2) = 5.88 \times 10^7$ N. The cross sectional area this time is a square, so $A = (0.5\text{ m})(0.5\text{ m}) = 0.25\text{ m}^2$. We can now use equation (8.3) again:

$$\frac{5.88 \times 10^7\text{ N}}{0.25\text{ m}^2} = (1100 \times 10^9\text{ GPa})\frac{\Delta L}{10.0\text{ m}}$$

We solve for ΔL as before:

$$\Delta L = \frac{5.88 \times 10^7\text{ N}}{0.25\text{ m}^2}\left(\frac{10.0\text{ m}}{1100 \times 10^9\text{ GPa}}\right) = \boxed{0.002\text{ m}}$$

The diamond pillar only compresses 2 mm.

[a] I know, I know.

Stress and Strain

The words *tension* and *stress* come from the same Latin root. In physics *stress* is the amount of pulling force *per unit area*, while tension is just the pulling force. Stress is F/A.

Strain is how much an object stretches (as a percent) as a result of the stress or pulling force. Strain is $\Delta L/L$.

If you look at the equation (8.3), you can see that stress and strain are related by the Young's modulus. In the **elastic regime**, this equation correctly describes how stress and strain are related.

$$\underbrace{\frac{F}{A}}_{\text{stress}} = E\,\underbrace{\frac{\Delta L}{L}}_{\text{strain}}$$

There's a *linear* relationship here, just like Hooke's law. That's not an accident, it turns out. In the elastic regime, objects act like springs. Stretch them and they will return to their normal length.

But nothing remains elastic with infinite stretching. Eventually the material becomes damaged.

If you stretch an object *too much*, past what is called the **elastic limit**, the object will *not* return to its normal length. This is like your slinky after you were foolish enough to lend to your childhood friend; it got stretched out so much that it no longer wanted to return to its normal length. When this happens, you have stretched an object into the **plastic regime**. The *material has been permanently damaged.*

Eventually, the object "fails" or breaks. This happens when an object exceeds its **ultimate tensile strength (UTS).** So we say that:

An object "fails" or breaks when

$$\frac{F}{A} \geq \text{UTS} \tag{8.4}$$

ULTIMATE TENSILE STRENGTH

UTS

The ultimate stress an object can experience before breaking. It has units of pascals (Pa).

A common way to represent the relationship between stress and strain in various regimes is a **stress-strain diagram** (see Figure 8.14). As an object in the elastic regime, stress and strain are related by a straight line relationship. But when the substance is stretched into the plastic regime, the curve flattens: less and less force gives more and more stretch. Eventually the object fails when the stress on the substance exceeds the ultimate tensile stress (UTS).

We use equation (8.4) to answer questions about when something breaks. I'll show a quick example below.

Quick example: What force is needed to barely break a 1.0 cm-diameter steel cable (UTS = 841 MPa)?

We're using equation (8.4), but we'll replace the inequality with "=" because the question as for the force needed to *barely* break the steel cable. The cross-sectional area A is the area of a 1 cm-diameter circle: $A = \pi(0.005\text{ m})^2$.

$$\begin{aligned}\frac{F}{\pi(0.005\text{ m})^2} &= 841 \times 10^6\text{ Pa}\\ F &= \pi(0.005\text{ m})^2(841 \times 10^6\text{ Pa}) = \boxed{66{,}000\text{ N}}\end{aligned}$$

which is a lot. That would be like hanging the weight of a ~ 6500 kg bull African elephant from the cable.

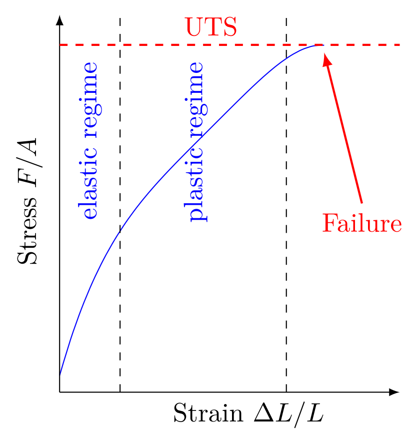


Figure 8.14: Stress strain diagram for a fiducial substance. The UTS for the substance is drawn as a horizontal, red dashed line.

Figure 8.15: Dragline spider silk, the type of silk from which spiders hang and the spokes of spider orb webs, is said to be stronger than steel. Credit: TGoeller, public domain, wikipedia.

"Stronger Than Steel"

You have probably heard that spider silk is "as strong as steel." This phrase is referring to the ultimate tensile strength of steel (841 MPa) and dragline spider silk (1.3 **GPa**). It takes a greater stress to break spider silk.

This demonstrates a worthwhile point about stress F/A. Stress already takes into account the size of the substance. The fact that spider silk breaks at a stress larger than the UTS of steel is saying "*given its size* spider silk would take greater force to break." Put another way, "if you could make a strand of steel the same size as spider silk, spider silk could withstand a larger force before breaking."

But this phrase is also a bit misleading. It doesn't mean that it requires more stress to *stretch it* then steel, which is usually what we think of when we think of *strength*. Spider silk ($E = 1.2$ GPa) stretches like crazy under relatively little stress, where steel ($E = 200$ GPa) simply won't.

So while spider silk is much more *stretchy* than steel, it requires more stress to *break*. And in this sense, it is "stronger than steel."

Quick example: Giant spiders are ubiquitous in movies. Calculate the force required to break a 1 cm-radius strand of dragline spider silk.

We're again using equation (8.4), The cross-sectional area A is the area of a 1 cm-diameter circle: $A = \pi(0.01\text{ m})^2$.

$$\begin{aligned}\frac{F}{\pi(0.01\text{ m})^2} &= 1.3\times10^9\text{ Pa}\\ F &= \pi(0.01\text{ m})^2(1.3\times10^9\text{ Pa}) = \boxed{408{,}000\text{ N}},\end{aligned}$$

enough to support 7 bull african elephants.

Review

8.1 Torque Equilibrium

1. Choose a pivot
2. Choose a positive rotational direction
3. Sum torques and set them equation to 0 N m

8.2 Balance

- An object is balanced when its center-of-mass is above its base of support
- Choose the pivot to be the balancing point or the fulcrum about which the system will tip

8.3 Springs

$$F_s = k\Delta x$$

8.4 Strength, Stress and Strain

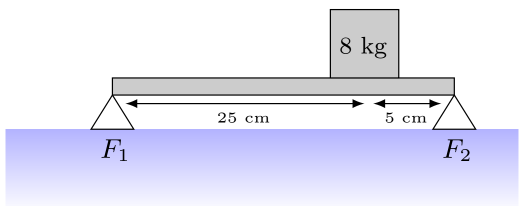
$$\frac{F}{A} = E\frac{\Delta L}{L}, \qquad \text{An object breaks if: } \frac{F}{A} \geq \text{UTS}$$

Problem Sets

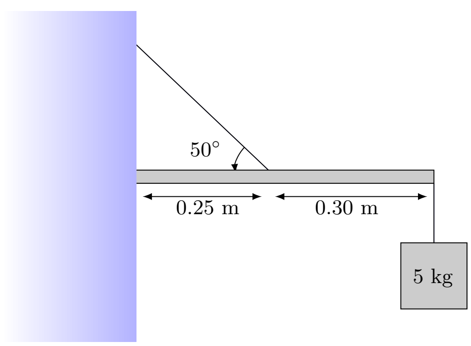
8.1 Torque and Equilibrium

Remember to sum *torques* for these problems to get your answers. Only summing forces in the y direction will lead to single equations with 2 unknowns which are not possible to solve.

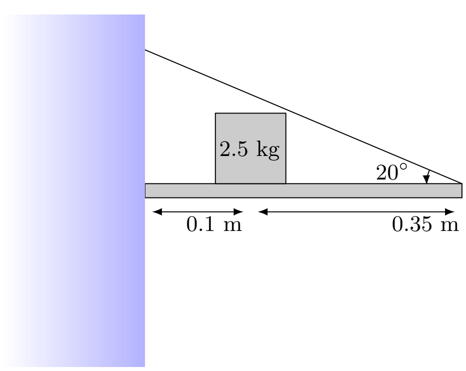
1. A 5.0 kg plank rests on two fulcrums as shown below. Find the upward force supplied by fulcrum F_1.



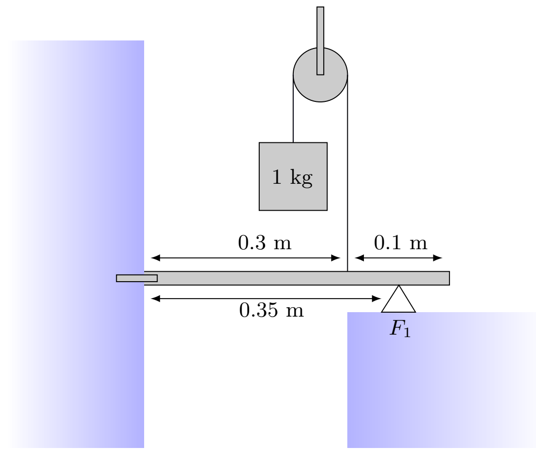
2. A 4.0 kg beam is attached to wall by a pivot. Find the tension T in the upper cable.



3. A 2.0 kg beam is attached to a wall by a pivot. What is the tension in the cable?



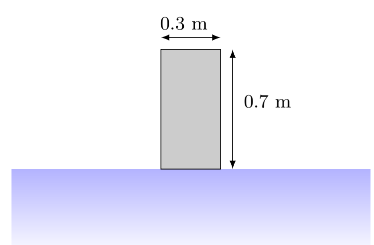
4. **Holy Ty Redd** An 8 kg beam is fixed to the left wall by a pivot. A 1 kg mass is attached to the beam via a rope strung over a pulley. Find the normal force provided by the fulcrum F_1.



This "hybrid" problem can be tackled exactly like the others. Use the pivot of the bar on the wall as your pivot and proceed as before.

8.2 Balance

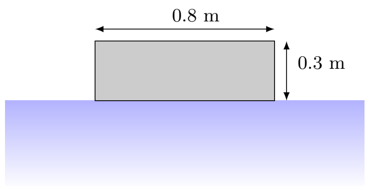
1. A block has the dimensions given in the figure below.



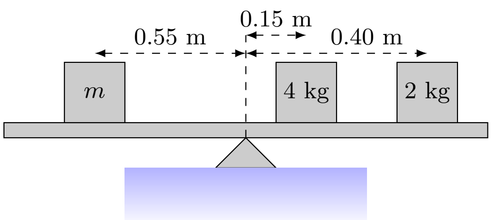
   (a) Draw a picture of the block at its critical angle,when block is tipped so that the center of mass is at the very edge of the supporting base.

   (b) What is the maximum angle that this block could be tilted before it tips over?

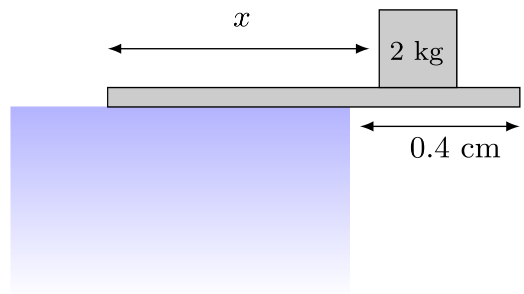
2. What is the maximum angle that this block could be tilted before it tips over? Compare this answer to your answer in question 1. Interpret your difference in your answers for critical angle in terms of the *stability* of the block.



3. What is the mass m which would balance the system? Assume the board is massless.

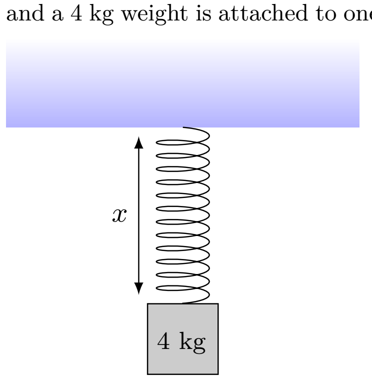


4. A 1 meter long, 2 kg board is placed on the edge of a table. How far from the base of the board may a 2 kg block be placed before the board tips over?



8.3 Springs

1. A spring with a natural length of 10 cm is *stretched* to the right to 15 cm. The spring constant k is 40.0 N/m. How much force does the spring exert? Which direction does it exert this force?
2. **2-Step Problem** It takes 10 N of force to stretch a spring by 0.02 m. How much force will it take to stretch the spring 0.06 m?
3. A very stiff spring has a spring constant $k = 150$ N/m and a natural length of 20 cm. What is the springs *new* length after it is hung from the ceiling and a 4 kg weight is attached to one end?



8.4 Strength of Materials

1. A 5.0 m-long steel circular wire ($E = 20 \times 10^{10}$ N m^2) with a diameter of 0.5 mm experiences a stretching force of 500 N. What is the change in length ΔL of the wire?

   Because I have given you a circular wire, the cross sectional area is a circle with area $A = \pi r^2$. Notice in the problem I have given you a diameter whereas the the area formula calls for radius, so we need to divide this by 2 before we can plug it in.

2. What force would be required to stretch a circular copper wire ($E = 11 \times 10^{10}$ N m^2) with a radius of 0.5 cm by 1.0 cm? The copper wire is originally 2.0 m long.

3. Some spider dragline silk has a diameter of 4.0 μm (4.0×10^{-6} m). It takes 0.065 N of force to break the dragline[3]. What is the tensile strength of spider dragline?

[3] These are "for-real" experimental values.